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STABILITY OF SPIRALLY STIFFENED SHELLS UNDER EXTERNAL PRESSURE

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Based on the use of a semizero-moment theory of thin shells, simple computational formulas for determination of the critical pressure on the shell have been obtained; these formulas contain all the necessary mechanical and geometric characteristics of the shell and the structure which are, in essence, a generalization of the well-known and widely used formulas for homogeneous isotropic shells.

Theoretical and experimental investigations have shown the efficiency of spiral stiffeners as compared to the stiffening with frames and stringers for certain types of loads [1–4]. Equations of the flat-shell theory have been used in [1]. In [3], it has been proposed that the equations of general theory, suitable for calculation of rising shells of any length, and (apparently, for the first time in analyzing the stability of shells) the equations of a semizero-moment theory of shells be used instead of the equations of flat shells.

Differential equilibrium equations are obtained by the method of variation of the total potential deformation energy; the deformation energy of stiffeners is averaged over the shell and involves tensile, flexural, and torsional deformations with allowance for eccentricity. Next, three differential equilibrium equations in translations are reduced by the operator method to a single equation for the normal translation; this equation differs from that in axial compression [5] only by the last term:

$$\Bigg\{c_{81}\frac{\partial^8}{\partial x^8}+c_{82}\frac{\partial^8}{\partial x^6\partial y^2}+c_{83}\frac{\partial^8}{\partial x^4\partial y^4}+c_{84}\frac{\partial^8}{\partial x^2\partial y^6}+c_{85}\frac{\partial^8}{\partial y^8}+c_{61}\frac{\partial^6}{\partial x^6}+c_{63}\frac{\partial^6}{\partial x^4\partial y^2}+c_{65}\frac{\partial^6}{\partial x^2\partial y^4}+$$
$$+c_{67}\frac{\partial^6}{\partial y^6}+c_{41}\frac{\partial^4}{\partial x^4}+c_{42}\frac{\partial^4}{\partial x^2\partial y^2}+c_{43}\frac{\partial^4}{\partial y^4}+j\frac{\partial^2}{\partial x\partial y}\Bigg(e_{81}\frac{\partial^6}{\partial x^6}+e_{82}\frac{\partial^6}{\partial x^4\partial y^2}+e_{83}\frac{\partial^6}{\partial x^2\partial y^4}+e_{84}\frac{\partial^6}{\partial y^6}+$$
$$+e_{61}\frac{\partial^4}{\partial x^4}+e_{62}\frac{\partial^4}{\partial x^2\partial y^2}+e_{63}\frac{\partial^4}{\partial y^4}+e_{41}\frac{\partial^2}{\partial x^2}+e_{42}\frac{\partial^2}{\partial y^2}\Bigg)+\Bigg[d_{11}\frac{\partial^4}{\partial x^4}+d_{13}\frac{\partial^4}{\partial x^2\partial y^2}+d_{15}\frac{\partial^4}{\partial y^4}+$$
$$+j\frac{\partial^2}{\partial x\partial y}\left(d_{12}\frac{\partial^2}{\partial x^2}+d_{14}\frac{\partial^2}{\partial y^2}\right)\Bigg]N_y\left(\frac{\partial^2}{\partial y^2}+1\right)\Bigg\}w=0\,. \quad (1)$$

All the notation adopted in [5] holds true here.

It is noteworthy that, for a simple one-dimensional spiral, we have $j = \sin 2\theta$ in Eq. (1), and the quantities containing rigidities in the coefficients must be divided in two. For a pair-symmetric spiral, we have $j = \sin 2\theta + \sin(-2\theta) = 0$.

In calculating shells of moderate and large length, we use Vlasov's semizero-moment theory of shells [6], obtaining the corresponding equations as a result of the application of the strong inequality

$$\partial^2 f/\partial y^2 >> \partial^2 f/\partial x^2\,, \quad (2)$$

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where f is any force or deformation factor in the shell, to the complete equations.

Carrying out the simplifications of resolvent (1), which follows from condition (2), we obtain

$$\left[c_{85}\frac{\partial^8}{\partial y^8}+c_{67}\frac{\partial^6}{\partial y^6}+c_{43}\frac{\partial^4}{\partial y^4}+c_{41}\frac{\partial^4}{\partial x^4}+d_{15}\frac{\partial^2}{\partial y^2}N_y\frac{\partial^2}{\partial y^2}\left(\frac{\partial^2}{\partial y^2}+1\right)\right]w=0\,. \tag{3}$$

In Eq. (3), for the case where the shell bulges to form long waves, we have allowed for the influence of circular translations on the potential.

For pair-symmetric spiral stiffeners (j = 0) we seek the solution of (1) in a form corresponding to the asymmetric form of stability loss:

$$w=A\sin\lambda_m x\cos ny\,,\quad \lambda_m=m\pi R/L\,. \tag{4}$$

For practical calculations it is important to have simple approximate formulas which would enable us to evaluate stability in the process of designing. Such a possibility is provided by the use of a semizero-moment theory of shells [3], just as in solving problems on the stressed-deformed state of shells in the case of extremum localized force and temperature actions [8]. In this case, by solution of Eq. (3) in the form (4) we obtain the expression for determination of the critical load under external pressure:

$$N_y=\frac{(n^2-1)\,c_{85}}{d_{15}}+\frac{\lambda_m^2 c_{41}}{n^4\,(n^2-1)\,d_{15}}\,. \tag{5}$$

From (5), it is seen that different values of m and n correspond to certain values of N_y. The stability loss will occur when the value of N_y is the minimum possible of all values. The quantity m appears only in the numerator (in terms of λ_m); therefore, it is clear that the condition of minimum in m will be fulfilled if we set m = 1 in (5). The quantity n appears in the numerator and the denominator; therefore, its influence on N_y must be examined additionally. Thus, for shells of moderate length $(3\sqrt{h/R}<L/R<3\sqrt{R/h})$ the wave number is usually $n\geq 4$; therefore, disregarding unity as compared to n^2 and minimizing Eq. (5) in n^2, we can obtain the computational formula in closed form:

$$N_y=\frac{4c_{85}}{3d_{15}}\sqrt{\frac{3\lambda_m^4 c_{41}}{c_{85}}}\,. \tag{6}$$

For shells of large length $(L/R>3\sqrt{r/h})$, when the second term in Eq. (5) can be disregarded, we have

$$N_y=\frac{(n^2-1)\,c_{85}}{d_{15}}=\frac{3c_{85}}{d_{15}}\,. \tag{7}$$

Disregarding terms of the order h/R as compared to unity in the expressions for the coefficients, for shells with pair-symmetric stiffeners (j = 0) we easily obtain formulas from relations (6) and (7) in a form convenient for practical calculations.

For shells of moderate and large length, we respectively find

$$p_{\mathrm{cr}}=\frac{0.835}{(1-\nu)^{3/4}}\frac{Eh^2}{LR}\sqrt{\frac{h}{R}}\sqrt[4]{k_1k_2^3}\,,\quad p_{\mathrm{cr}}=\frac{3D}{R^3}k_2\,.$$

Here

$$k_1=\frac{1+\dfrac{2A_{\mathrm{r}}}{hl_{\mathrm{r}}}\left(1-\dfrac{1+v}{2}\sin^2 2\theta\right)}{1+\dfrac{2\,(1-v^2)\,A_{\mathrm{r}}\sin^4\theta}{hl_{\mathrm{r}}}}\,;\quad k_2=1+\frac{12\,(1-v^2)}{h^3}\left[\frac{J_{\mathrm{r}}\sin^2 2\theta}{4\,(1+v)\,l_{\mathrm{r}}}+\frac{2J'_{\mathrm{s.r}}\sin^4\theta}{l_{\mathrm{r}}}\right],$$

where $J'_{\mathrm{s.r}} = J_\eta + \dfrac{\bar{z}^2 A_\mathrm{r}}{\left[1 + \dfrac{2(1-\nu^2)\, A_\mathrm{r} \sin^4\theta}{h l_\mathrm{r}}\right]}$ is the rib's moment of inertia computed with allowance for the joint work with the skin's belt. Hence we obtain the classical formula of Papkovich for a smooth shell and the formula for a long shell [9]

$$p_{\mathrm{cr}} = 0.92 \cdot \frac{Eh^2}{LR} \sqrt{\frac{h}{R}} \,, \quad p_{\mathrm{cr}} = 3D/R^3 \,.$$

Numerical results of the investigations have been shown that, unlike axial compression [5], where the advantage of spiral stiffeners over regular ones is preserved for shells of any length, the efficiency of spiral stiffeners is reduced under the action of external pressure on the structure with increase in the shell length; for very long shells it is more expedient to use stiffeners in the form of frames.

NOTATION

A, amplitude value of translation, m; A_r, cross-sectional area of the rib, m^2; D, cylindrical rigidity, N·m; E, elastic modulus of the shell material, N/m^2; f, any force or deformation factor in the shell; h, shell thickness, m; J_η and J_z, moments of inertia of the cross section of a spiral rib, m^4; L, shell length, m; l_r, distance along the normal between two neighboring parallel spirals, m; m, harmonic No. in the longitudinal direction of the shell; n, number of total waves in the circular direction; N_y, circular force, N/m; p, external normal pressure, N/m^2; p_{cr}, value of the normal pressure, N/m^2; R, shell radius, m; w, normal translation, m; x, y, z, dimensionless coordinate system of the shell (longitudinal, circular, and normal coordinate axes); $\bar{z}$, distance from the median surface of the shell to the center of gravity of the rib, m; ν, Poisson coefficient of the shell material; θ, slope of spiral stiffeners, deg; ξ, η, dimensionless coordinate system of the cross section of the rib. Subscripts: cr, critical; s, skin; s.r, skin's belt with a rib relative to the median of the skin; z and η, coordinate axes.

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